Nonautonomous solitons of Bose-Einstein condensation in a linear potential with an arbitrary time-dependence

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In the presence of a linear potential with an arbitrary time-dependence, Hirota method is developed carefully for applying into the effective mean-field model of quasi-one-dimensional Bose-Einstein condensation with repulsive interaction. We obtain the exact nonautonomous soliton solution (NSS) analytically. These solutions show that the time-dependent potential can affect the velocity of NSS. In some special cases the velocity has the character of both increase and oscillation with time. A detail analysis for the asymptotic behaviour of solutions shows that the collision of two NSSs is elastic.

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I. INTRODUCTION

The concept of soliton was introduced firstly by Zabusky and Kruskal [1] to characterize nonlinear solitary waves that do not disperse and completely preserve their localized form and speeds during propagation and after a collision. This intrinsic favorable property of soliton has motived a great of attention on the nonlinear systems in many fields of physics, especially in high-rate telecommunications with optical fibers and condensate physics. Hasegawa and Tappert [2] derived the nonlinear Schrödinger (NLS) equation model in fiber and firstly predicted optical soliton, and then experimental verification has successfully been carried out by Mollenauer et al. [3]. Since then, optical solitons have been the objects of extensive theoretical and experimental studies in the past three decades for their potential applications in long distance communication and all-optical ultrafast switching devices. Recently, the controllable soliton solutions [4–7] are of interest in the field of nonlinear optics and condensate physics, and then the term of nonautonomous solitons [6] introduced firstly. In fact, different aspects of dynamics in nonautonomous models [8, 9] in linear potentials have been investigated theoretically. Strictly speaking, this nonautonomous solutions obtained could not be considered as canonical solitons. Fortunately, the realization of the Bose-Einstein condensation (BEC) [10, 11] offered a good examples of the nonautonomous systems in condensate physics.

With the realization of BECs the exploration of the nonlinear properties of matter waves has been paid more particular interest. One of them is macroscopically excited BECs, such as vortices [12] and solitons [13–19]. At zero temperature the dynamics of BEC is well described by the time-dependent Gross-Pitaevskii (G-P) equation, and the nonlinearity results from the interatomic interactions. Depending on the attractive or repulsive nature of the interatomic interactions, G-P equation has of either bright or dark soliton solutions, respectively. A bright soliton [20–22] in BEC is expected for the balance between the dispersion and the attractive mean-field energy. However, large condensates are necessarily associated with repulsive interaction, for which bright soliton might seem impossible because the nonlinearity cannot compensate for the kinetic energy part in the atomic dynamics. So it is interesting to explore the property for dark soliton of BEC. A dark soliton [23, 24] in BEC is a macroscopic excitation characterized by a local density minimum and a phase gradient of the wave function at the position of the minimum. Under the different conditions many soliton solutions [15–19, 25, 26] have been obtained, as well as the dynamics of the excitation of the condensate was discussed. When the longitudinal dimension of BEC is much longer than its transverse dimension which is the order of its healing length, the G-P equation can be reduced to the quasi-one-dimensional (quasi-1D) regime. This trapped quasi-one-dimensional [27] condensate has offered an useful tool to investigate the nonlinear excitations such as solitons and vortices, which are more stable than in 3D, where the solitons suffer from the transverse instability and the vortices can bend. So the studies of both theory and experiment are very important for the soliton excitation in quasi-one-dimensional BEC.

The effective mean-field model of a quasi-1D BEC in a linear potential with an arbitrary time-dependence is given by

$$i\hbar\frac{\partial}{\partial T}\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial X^2}\Psi + Xf(T)\Psi + g\left|\Psi\right|^2\Psi, \qquad (1)$$

where $\int |\Psi|^2 dX = N$ is the number of atoms in the condensate. The interacting constant of two-atom is $g = 2\hbar^2 a/ml_0^2$ [28], where m is the mass of the atom, a is the s-wave scattering length ($a < 0$ for attractive interaction; while $a > 0$ for repulsive interaction), and $l_0 \equiv \sqrt{\hbar/m\omega_0}$ is the characteristic extension length of the ground state wave function of harmonic oscillator. For pithiness, we introduce $x = X/l_0$, $t = T/ml_0^2/\hbar$, and $\psi = \Psi/\sqrt{Nl_0}$,

and then Eq. (1) reduces to the dimensionless form

$$i\frac{\partial}{\partial t}\psi+\frac{1}{2}\frac{\partial^2}{\partial x^2}\psi+xf\left(t\right)\psi-\mu\left|\psi\right|^2\psi=0, \tag{2}$$

where $\mu = 2Nl_0a$, and $f\left(t\right) = -\frac{ml_0^3}{\hbar^2}f(\frac{T}{\hbar/ml_0^2})$. As in Ref. [6], Eq. (2) was called nonautonomous NLS model in linear potential. When $f\left(t\right) = constant$ and $a < 0$, the Lax pair and NSSs in inhomogeneous plasma has been constructed by the inverse scattering method [8]. F-expansion method [29] and Hirota method [30] were also developed to construct the bright NSSs in quasi-1D BECs. However, the dynamic property of NSS of Eq. (2) hasn't well explored, and Hirota method developed for Eq. (2) with the repulsive interaction is also very interesting.

In the present paper we consider mainly the dynamics of dark nonautonomous soliton which can be affected by adjusting the external linear time-dependent potential. In the following section we demonstrate how to construct the exact dark NSSs of Eq. (2), and the corresponding properties of such solutions are studied in detail.

II. DEVELOPED HIROTA METHOD AND ONE NONAUTONOMOUS SOLITON SOLUTION

Hirota method [31] is an effective straightforward technique to solve the nonlinear equations. In order to clear the derivation of solution we introduce the main idea of Hirota method briefly. Firstly, it apply a direct transformation to the nonlinear equation. Then, by means of some skillful bilinear operators the nonlinear equation can be decoupled into a series of equations. With some reasonable assumptions the exact solutions can be constructed effectively. However, in the presence of the time-dependent potential the application of Hirota method should be more careful to get NSSs of Eq. (2) in the case of the repulsive interaction.

Performing the normal procedure, we consider the complex function $G\left(x,t\right)$ and the real function $F\left(x,t\right)$ forming the transformation

$$\psi=\frac{G(x,t)}{F(x,t)}. \tag{3}$$

Substituting Eq. (3) into Eq. (2) we have

$$F(iD_t+\frac{D_x^2}{2})G\cdot F+GF^2xf(t)-G(\frac{D_x^2}{2}F\cdot F+\mu\overline{G}G)=0, \tag{4}$$

where the overbar denotes the complex conjugate, D_t and D_x^2 are called Hirota bilinear operators defined by

$$\begin{aligned}&D_x^mD_t^nG\left(x,t\right)\cdot F\left(x',t'\right)\\&=(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'})^m(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'})^nG\left(x,t\right)F\left(x',t'\right)\Big|_{x=x',t=t'}.\end{aligned} \tag{5}$$

In the absence of the time-dependent potential, i.e., $xf(t) = 0$, Eq. (4) can be decoupled easily into two equations. In order to get exact dark NSSs of Eq. (2), a real parameter λ to be determined should be added to Hirota bilinear operators in the presence of the time-dependent potential, and then many attempts show that Eq. (4) can be decoupled into

$$\hat{\mathcal{A}}_1G\cdot F=0,\hat{\mathcal{A}}_2F\cdot F=-\mu G\overline{G}, \tag{6}$$

where the overbar denotes the complex conjugate, and Hirota bilinear operators $\hat{\mathcal{A}}_1$ and $\hat{\mathcal{A}}_2$ are given by

$$\begin{aligned}\hat{\mathcal{A}}_1&=iD_t+\frac{1}{2}D_x^2+xf(t)-\lambda,\\\hat{\mathcal{A}}_2&=\frac{1}{2}D_x^2-\lambda.\end{aligned}$$

The derivation of Eq. (6) has made Eq. (2) into the normal procedure of Hirota method. The spatial and time dependence term $xf(t)$ will play an important role for getting the exact NSSs as shown later.

If the expressions of $G\left(x,t\right)$ and $F\left(x,t\right)$ are obtained from Eq. (6), the exact dark NSSs can be expressed analytically. For this purpose we assume that

$$G=G_0\left(1+\chi G_1\right),F=\left(1+\chi F_1\right), \tag{7}$$

where χ is an arbitrary auxiliary parameter which will be absorbed in the expression of NSSs. Substituting Eq. (7) into Eq. (6), and collecting the coefficients with same power of χ, we have

(1) for the coefficient of χ^0

$$\hat{\mathcal{A}}_1\left(G_0\cdot 1\right)=0, \tag{8}$$

$$\hat{\mathcal{A}}_2\left(1\cdot 1\right)=-\mu G_0\overline{G}_0, \tag{9}$$

(2) for the coefficient of χ^1

$$\hat{\mathcal{A}}_1\left(G_0G_1\cdot 1+G_0\cdot F_1\right)=0, \tag{10}$$

$$\hat{\mathcal{A}}_2\left(F_1\cdot 1+1\cdot F_1\right)=-\mu G_0\overline{G}_0\left(G_1+\overline{G}_1\right), \tag{11}$$

(3) for the coefficient of χ^2

$$\hat{\mathcal{A}}_1\left(G_0G_1\cdot F_1\right)=0, \tag{12}$$

$$\hat{\mathcal{A}}_2\left(F_1\cdot F_1\right)=-\mu G_0\overline{G}_0G_1\overline{G}_1. \tag{13}$$

Using the definition of Hirota bilinear operator (5) the above equations can be expressed in detail. Considering the presence of the term $xf(t)$ in Eq. (8) we assume G_0 has the form

$$G_0=\gamma_0e^{i\eta_0}, \tag{14}$$

where

$$\eta_0=P_0(t)x+\Omega_0(t), \tag{15}$$

with $P_0(t)$ and $\Omega_0(t)$ is to be determined, respectively. Substituting G_0 into Eq. (8) we have

$$0=\left[-P_{0,t}(t)+f(t)\right]x-\Omega_{0,t}(t)-\frac{1}{2}P_0^2(t)-\lambda,$$

which implies the solution

$$P_0(t) = \int_0^t f(\tau)\,d\tau + \xi_0,$$
$$\Omega_0(t) = -\frac{1}{2}\int_0^t P_0^2(\tau)\,d\tau - \lambda t + \zeta_0, \qquad (16)$$

where ξ_0 and ζ_0 is an arbitrary real constant, respectively. From the restriction of Eq. (9) we get $|\gamma_0|^2 = \lambda/\mu$ which shows that the exist of dark NSS demand the parameter $\lambda > 0$. For convenience γ_0 can be chosen as $\gamma_0 = \sqrt{\lambda/\mu}$.

Expanding Eqs. (10) and (11) with the definition of Eq. (5) one can see that G_1 and F_1 admit the expression

$$G_1 = Z_1 \exp\eta_1, F_1 = \exp\eta_1, \qquad (17)$$

where the parameter Z_1 to be determined is complex, and the real parameter η_1 is given by

$$\eta_1 = P_1(t)\,x + \Omega_1(t). \qquad (18)$$

Substituting Eqs. (14) and (17) into Eq. (10) we have

$$0 = i(Z_1 - 1)(P_{1,t}x + \Omega_{1,t} + P_0P_1) + \frac{1}{2}(Z_1 + 1)P_1^2. \qquad (19)$$

In the case of $Z_1 \neq \pm1$(the case of $Z_1 = \pm1$ will be discussed in below), the solution of Eq. (19) is $P_{1,t} = 0$, i.e., P_1 should be independent on t, and Z_1 is given by

$$Z_1 = \frac{iP_0P_1 - \frac{1}{2}P_1^2 + i\Omega_{1,t}}{iP_0P_1 + \frac{1}{2}P_1^2 + i\Omega_{1,t}}, \qquad (20)$$

which shows that $|Z_1|^2 = 1$. From Eqs. (11) to (13) we get the restriction

$$P_1^2 = 2\lambda - \lambda\left(Z_1 + \overline{Z}_1\right). \qquad (21)$$

Then from Eq. (20) and Eq. (21) we obtain

$$Z_1 = \frac{\sqrt{4\lambda - P_1^2} + iP_1}{\sqrt{4\lambda - P_1^2} - iP_1},$$
$$\Omega_1 = -P_1\int_0^t P_0(\tau)\,d\tau + \frac{1}{2}\sqrt{4\lambda - P_1^2}P_1 t + \zeta_1, \qquad (22)$$

where ζ_1 is a real constant, P_1 is an arbitrary real parameter, and $\lambda \geq P_1^2/4$.

With the help of Eqs. (7), (14) and (17), after absorbing χ, the exact dark NSS of Eq. (2) can be derived as

$$\psi_1 = \frac{1}{2}\sqrt{\frac{\lambda}{\mu}}e^{i\eta_0}\left[(1 + Z_1) - (1 - Z_1)\tanh\frac{\eta_1}{2}\right], \qquad (23)$$

where the parameters η_0, η_1, and Z_1 have been given in Eqs. (15), (16), (18) and (22).

If one chose $\lambda = 4\eta^2$ and $P_1 = 4a_1\eta^2$, the solution in Eq. (23) has the same form as the general solution obtained in the framework [6]. In the case of $f(t) = 0$, the solution in Eq. (23) reduces to the one soliton solution of the normal NLS equation. When $f(t) =$*constant*, the solution (23) represents the nonlinear dark wave propagation in linearly inhomogeneous plasma [8] or optical fibre with the abnormal dispersion. As $f(t) = b_1 + l\cos(\omega t)$, the solution (23) denotes NSS in BEC with considering the coupling of the external field and the effect of gravity.

From the NSS in Eq. (23) we clear two special case mentioned before, i.e., $Z_1 = \pm1$. When $Z_1 = 1$, the solution (23) reduces to plane-wave solution $\psi_1 = \sqrt{\lambda/\mu}e^{i\eta_0}$, which corresponds to the uniform distribution density of bosons. On the other hand, when $Z_1 = -1$ the solution (23) becomes $\psi_1 = -\sqrt{\lambda/\mu}e^{i\eta_0}\tanh(\eta_1/2)$, where $\eta_1 = 2\sqrt{\lambda}x - 2\sqrt{\lambda}\int_0^t P_0(\tau)\,d\tau + \zeta_1$. This solution represents the black NSS of BEC in a linear potential with an arbitrary time-dependence which is caused by the coupling of external field and the effect of gravity.

We also find the effect of term $xf(t)$ from the NSS in Eq. (23). As shown in the expression of η_0, the term $xf(t)$ can only contribute a phase to the background. The width of nonautonomous soliton, defined by $1/P_1$, is not affected by the time-dependent external potential. From Eqs. (18) and (22) we get the velocity

$$V_1 = -\frac{\partial}{\partial t}\frac{\Omega_1}{P_1} = \int_0^t f(\tau)\,d\tau + \xi_0 - \frac{1}{2}\sqrt{4\lambda - P_1^2},$$

which shows that the time-dependent potential play an important role for the velocity of dark NSS. When $f(t) = b_1 + l\cos(\omega t)$, the velocity becomes $V_1 = b_1 t + l/\omega\sin(\omega t) + \xi_0 - \sqrt{\lambda - (P_1/2)^2}$, which increases and oscillates with time. These results shows that under the effect of gravity, the BEC slides down where the dark nonautonomous soliton slides down and oscillates in time. It can be realized by controlling the external field.

III. COLLISION OF TWO DARK NONAUTONOMOUS SOLITONS

In this section we will give the analytical expression of two dark NSSs of Eq. (2). In this case $G(x,t)$ and $F(x,t)$ of Eq. (3) are assumed as

$$G = G_0\left(1 + \chi G_1 + \chi^2 G_2\right), F = 1 + \chi F_1 + \chi^2 F_2. \qquad (24)$$

where G_0 has been obtained in Eq. (14). Employing the similar procedure of the above section we obtain the following set of equations from Eq. (6)

(1) for the coefficient of χ

$$\pounds_1(G_1 \cdot 1 + 1 \cdot F_1) = 0, \qquad (25)$$
$$\hat{\mathcal{A}}_2(1 \cdot F_1 + F_1 \cdot 1) = -\lambda\left(G_1 + \overline{G}_1\right), \qquad (26)$$

(2) for the coefficient of χ^2

$$\pounds_1(1 \cdot F_2 + G_1 \cdot F_1 + G_2 \cdot 1) = 0, \qquad (27)$$
$$\hat{\mathcal{A}}_2(1 \cdot F_2 + F_1 \cdot F_1 + F_2 \cdot 1) = -\lambda\left(G_2 + \overline{G}_1 G_1 + \overline{G}_2\right), \qquad (28)$$

(3) for the coefficient of χ^3

$$\pounds_1 \left(G_1 \cdot F_2 + G_2 \cdot F_1\right) = 0, \quad (29)$$

$$\hat{\mathcal{A}}_2 \left(F_1 \cdot F_2 + F_2 \cdot F_1\right) = -\lambda \left(\overline{G}_2 G_1 + \overline{G}_1 G_2\right), \quad (30)$$

(4) for the coefficient of χ^4

$$\pounds_1 G_2 \cdot F_2 = 0, \quad (31)$$

$$\hat{\mathcal{A}}_2 F_2 \cdot F_2 = -\lambda \overline{G}_2 G_2, \quad (32)$$

where $\pounds_1 = iD_t + \frac{1}{2}D_x^2 + P_0(t)D_x$, and $\hat{\mathcal{A}}_2$ is given before.

It is obvious that one can solve the equations (25) to (32) in turn with the reasonable expressions of G_1 and F_1. A detail analysis shows that G_1 and F_1 admit the forms

$$G_1 = Z_1 \exp \eta_1 + Z_2 \exp \eta_2, F_1 = \exp \eta_1 + \exp \eta_2, \quad (33)$$

where Z_j is complex and η_j has the form

$$\eta_j = P_j\left(t\right) x + \Omega_j\left(t\right), j = 1, 2, \quad (34)$$

with the parameter $P_j(t)$ and $\Omega_j(t)$ is to be determined, respectively. Substituting Eq. (33) into Eq. (25) we have

$$\begin{aligned} 0 = e^{\eta_1}[(Z_1 - 1)(iP_{1,t}x + iP_0 P_1 + i\Omega_{1,t}) + \frac{Z_1 + 1}{2} P_1^2] \\ + e^{\eta_2}[(Z_2 - 1)(iP_{2,t}x + iP_0 P_2 + i\Omega_{2,t}) + \frac{Z_2 + 1}{2} P_2^2]. \end{aligned}$$

In the case of $Z_1 \neq \pm 1$ and $Z_2 \neq \pm 1$, the above equation implies that $P_{j,t}\left(t\right) = 0$, $j = 1, 2$, i.e., P_j is independent on t, and Z_j is given by

$$Z_j = \frac{iP_0 P_j - \frac{1}{2}P_j^2 + i\Omega_{j,t}}{iP_0 P_j + \frac{1}{2}P_j^2 + i\Omega_{j,t}}, \quad (35)$$

which shows that $\left|Z_j\right|^2 = 1$, $j = 1, 2$. From Eq. (26) we have the restriction

$$P_j^2 = 2\lambda - \lambda\left(Z_j + \overline{Z}_j\right), j = 1, 2. \quad (36)$$

From Eqs. (35) and (36) we have

$$\begin{aligned} Z_j &= \frac{\sqrt{4\lambda - P_j^2} + iP_j}{\sqrt{4\lambda - P_j^2} - iP_j}, \\ \Omega_j &= -P_j \int_0^t P_0\left(\tau\right) d\tau + \frac{1}{2}\sqrt{4\lambda - P_j^2} P_j t + \zeta_j, \end{aligned} \quad (37)$$

where ζ_j is a real constant, P_j is an arbitrary real parameter, and $\lambda \geq P_j^2/4$, $j = 1, 2$.

Substituting Eq. (33) into Eqs. (27) and (28), after a tedious and expatiatory calculation we obtain the expressions of G_2 and F_2 as

$$G_2 = A_{12} Z_1 Z_2 \exp\left(\eta_1 + \eta_2\right), F_2 = A_{12} \exp\left(\eta_1 + \eta_2\right), \quad (38)$$

where the real parameter A_{12} is given by

$$A_{12} = \frac{4\lambda - P_1 P_2 - \sqrt{4\lambda - P_1^2}\sqrt{4\lambda - P_2^2}}{4\lambda + P_1 P_2 - \sqrt{4\lambda - P_1^2}\sqrt{4\lambda - P_2^2}}.$$

Now we have obtained the expression of G_0, G_1, G_2, F_1 and F_2 in Eq. (24). With the help of Eqs. (33) to (38) one can find the Eqs. (29) to (32) are satisfied to the moment.

With Eqs. (3), (14), (33), and (38), while absorbing the parameter χ, we obtain the dark two NSS of Eq. (2) as

$$\psi_2 = \sqrt{\frac{\lambda}{\mu}} e^{i\eta_0} \frac{1 + Z_1 e^{\eta_1} + Z_2 e^{\eta_2} + A_{12} Z_1 Z_2 e^{\eta_1 + \eta_2}}{1 + e^{\eta_1} + e^{\eta_2} + A_{12} e^{\eta_1 + \eta_2}}. \quad (39)$$

When $f(t) = 0$, the solution (39) denotes dark two solitons interaction of the normal NLS equation. When $f(t) =$*constant*, the solution (39) represents the dynamics of two nonautonomous nonlinear waves in linearly inhomogeneous plasma or optical fibre with the abnormal dispersion. As shown before, the expressions (23) and (39) imply that Hirota method has more advantage for getting such solutions as well.

The solution in Eq. (39) describes a general scattering process of two dark NSSs with different center velocity V_1 and V_2, respectively. From Eqs. (34) and (37) we get each velocity as

$$V_j = \int_0^t f\left(\tau\right) d\tau + \xi_0 - \frac{1}{2}\sqrt{4\lambda - P_j^2}, j = 1, 2.$$

Under the proper parameters two NSSs can move toward each other, one with the velocity V_1, while the other with V_2. In order to understand the nature of two NSSs interaction, we analyze the asymptotic behave of the solution in Eq. (39). Asymptotically, the solution in Eq. (39) can be written as a combination of two NSSs in Eq. (23). The asymptotic form of two NSSs in limits $t \to -\infty$ and $t \to \infty$ is similar to that of the one NSS in Eq. (23).

(i) Before collision (limit $t \to -\infty$)

(a) Nonautonomous soliton 1 ($\eta_1 \approx 0$, $\eta_2 \to -\infty$)

$$\psi_2 \to \frac{1}{2}\sqrt{\frac{\lambda}{\mu}} e^{i\eta_0} \left[(1 + Z_1) - (1 - Z_1)\tanh \frac{\eta_1}{2}\right], \quad (40)$$

(b) Nonautonomous soliton 2 ($\eta_2 \approx 0$, $\eta_1 \to \infty$)

$$\psi_2 \to \frac{1}{2}\sqrt{\frac{\lambda}{\mu}} Z_1 e^{i\eta_0} \left[(1 + Z_2) - (1 - Z_2)\tanh \frac{1}{2}\left(\eta_2 + \delta_0\right)\right]. \quad (41)$$

(ii) After collision (limit $t \to \infty$)

(a) Nonautonomous soliton 1 ($\eta_1 \approx 0$, $\eta_2 \to \infty$)

$$\psi_2 \to \frac{1}{2}\sqrt{\frac{\lambda}{\mu}} Z_2 e^{i\eta_0} \left[(1 + Z_1) - (1 - Z_1)\tanh \frac{1}{2}\left(\eta_1 + \delta_0\right)\right], \quad (42)$$

(b) Nonautonomous soliton 2 ($\eta_2 \approx 0$, $\eta_1 \to -\infty$)

$$\psi_2 \to \frac{1}{2}\sqrt{\frac{\lambda}{\mu}}e^{i\eta_0}\left[(1+Z_2)-(1-Z_2)\tanh\frac{\eta_1}{2}\right], \quad (43)$$

where the center shift is given by $\delta_0 = \ln A_{12}$. By analyzing the asymptotic behave in detail we know that there is no change of the amplitude for each NSS during collision, while one should notice that the factor $|Z_j| = 1$, $j = 1, 2$, again. However, from Eq. (40) to Eq. (43) we find a phase exchange δ_0 for soliton 1 and soliton 2 during collision. These results show that the collision of two NSSs is elastic.

IV. CONCLUSION

In this paper, we report the exact dark NSSs of quasi-one-dimensional BEC in a linear potential with an arbitrary time-dependence, and Hirota method is also developed. With the skillful assumption the exact dark NSSs are constructed effectively. From these results we find the time-dependent potential can affect the velocity of NSS. In some special cases the velocity of NSS in quasi-one-dimensional BEC increases and oscillations in time, the BEC slides down under the effect of gravity. We also investigate the asymptotic behave of two NSSs which denotes the elastic collision.

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